POLITEHNICA UNIVERSITY TIMIŞOARA

Faculty of Electronics and Telecommunications

Studies in English

BACHELOR FINAL EXAM

Academic Year 2015-2016

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Mathematics

1. *Present Taylor's formula for functions of one variable and how can be used in approximating functions by polynomials.*

Answer:

Let $f : I \subset \mathbf{R} \to \mathbf{R}$, and $x_0 \in I$, where $f \in C^{n+1}(I)$. Then $f(x) = T_n(x) + R_n(x)$ (*Taylor's formula*),

where T_n is the Taylor's polynomial of nth order, and R_n is the reminder:

$$T_n(x) = f(x_0) + \frac{x - x_0}{1!} f'(x_0) + \ldots + \frac{(x - x_0)^n}{n!} f^{(n)}(x_0),$$

$$R_n(x) = \frac{(x - x_0)^{n+1}}{(n+1)!} f^{(n+1)}(x_0 + \theta(x - x_0)),\ 0 < \theta < 1.$$

It follows the approximation formula for $f(x)$ in a neighborhood V of x_0:

$$f(x) \cong T_n(x),$$

with the error $\varepsilon_n = \sup_{x \in V} |R_n(x)|$.

2. *Define the notions of eigenvalue (or proper value) and eigenvector (or proper vector) on a linear operator.*

***Answer*:**

We consider the vector space V defined over the field **K** and the linear operator f: V → V. A vector v V (different from the null vector of V) is called an eigenvector (or proper vector) of the operator f if there exists a scalar λ from **K** such that $f(v) = \lambda v$. The scalar λ is called an eigenvalue (or proper value) of f.

3. *Specify how the extremes of a function of class C^2 of two variables can be found.*

Answer:

The extremes of the function $u = u(x, y)$ are among the *stationary points*, namely the solutions of the system $\begin{cases} \dfrac{\partial u}{\partial x} = 0 \\ \dfrac{\partial u}{\partial y} = 0 \end{cases}$.

A stationary point is a point of *minimum* if in this point

$$\frac{\partial^2 u}{\partial x^2} \cdot \frac{\partial^2 u}{\partial y^2} - \left(\frac{\partial^2 u}{\partial x \partial y} \right)^2 > 0 \text{ and } \frac{\partial^2 u}{\partial x^2} > 0,$$

and is a point of *maximum* if in this point

$$\frac{\partial^2 u}{\partial x^2} \cdot \frac{\partial^2 u}{\partial y^2} - \left(\frac{\partial^2 u}{\partial x \partial y} \right)^2 > 0 \text{ and } \frac{\partial^2 u}{\partial x^2} < 0.$$

4. *Define the following notions: arithmetical mean, weighted arithmetical mean and geometrical mean.*

***Answer*:**

Let $\{x_1, x_2, \ldots, x_n\}$ be a non-empty set of records (real numbers) with non-negative wedges $\{p_1, p_2, \ldots, p_n\}$.

Weighted mean: $M_p = \dfrac{p_1 x_1 + p_2 x_2 + \cdots + p_n x_n}{p_1 + p_2 + \cdots p_n}$ (the elements with a greater weight have more contribution to the mean). We can simplify the above formula taking normalized weights $\sum_{i=1}^{n} p_i = 1$. In this case we have $M_p = \sum_{i=1}^{n} p_i x_i$

Arithmetical mean : M_a it is a particular case of the weight mean M_p when all weights are equals $p_n = \frac{1}{n}$.

We have $M_a = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{x_1 + x_2 + \cdots + x_n}{n}$ (M_a indicates the central trend of a set numbers).

Geometrical mean : $M_g = \sqrt[n]{x_1, x_2, \cdots x_n}$ if $x_i > 0$, $i = \overline{1,n}$. The geometrical mean has the following geometric explanation: the geometrical mean $M_g = \sqrt{ab}$ of two numbers $a, b \in \mathbf{R}_+$ represents the length of a square with the same area as a rectangle with lengths a and b.

5. *Define the notion of the conditional probability, write and explain the Bayes's formula.*

***Answer*:**

Let $\{E, K, P\}$ a probability space and $A, B \in K$ two events with $P(A) \neq 0$. We call the probability of the event B conditioned by the event A, the expression:

$$P_A(B) = P(B / A) = \frac{P(A \cap B)}{P(A)}$$

Let $S = \{B_1, B_2 \ldots B_n\}$ an events complete system. Therefore, $E = \bigcup_{i=1}^{n} B_i, B_i \in K, B_i \bigcap B_j = \phi, i \neq j$. We say that the system S is a partition of the sure event E, and the events B_i are called outcomes.

Bayes's formula:

$$P_A(B_i) = \frac{P(B_i) \cdot P_{B_i}(A)}{\sum_{j=1}^{n} P(B_j) \cdot P_{B_j}(A)}$$

This formula returns the probability of an outcome in the hypothesis that the event A has occured, or, more precisely, the probability that to occur the event A to be conditioned by the outcome B_i.

6. Define for a discrete (and finite) random variable the following numerical characteristics: mean value, variance and standard deviation.

***Answer*:**

Let ξ be a discrete (and finite) random variable with its probability distribution

$$\xi : \begin{pmatrix} x_1, x_2, \ldots, x_n \\ p_1, p_2, \ldots, p_n \end{pmatrix}, \sum_{i=1}^{n} p_i = 1, \ p_i = P(\xi = x_i)$$

Mean value: $M(\xi) = \sum_{i=1}^{n} x_i \, p_i$. The mean value represents a numerical value around which it's find a group of the values for this random variable.

Variance: $D^2(\xi) = \sigma^2 = M\left[(\xi - M(\xi))^2\right]$.

Standard deviation: $D(\xi) = \sigma = \sqrt{D^2(\xi)}$.

The variance and the standard deviation are indicators which explain the "scattering" of the values for a random variable, giving information on the concentration degree of the values around to its mean value.

7. Define the Laplace transform and write the formula for the derivative.

***Answer*:**

If *f* is an original function, then its Laplace transform is

$$(Lf)\,(s) = \int_0^{\infty} f(t) e^{-st} dt$$

Image of the derivative: $(Lf')\,(s) = s\,(Lf)(s) - f(0_+)$.

8. Define the Z transform (the discrete Laplace transform) and calculate its image for the unit-step signal.

***Answer*:**

If $\{fn\}$ is an original sequence, then its Z transform is:

$$Z(f_n)\,(z) = \sum_{n=1}^{\infty} f_n z^{-n}.$$

For the unit-step signal

$$\sigma_n = \begin{cases} 0, & n < 0, \\ 1, & \mathrm{n} \geq 0, \qquad \mathrm{n} \in \mathbb{Z} \end{cases}$$

its Z transform is

$$Z(\sigma_n)\,(z) = \sum_{n=1}^{\infty} z^{-n} = \frac{1}{1-\frac{1}{z}} = \frac{z}{z-1}, \quad \text{for} \quad |\,z\,| < 1.$$

9. Polar, cylindrical and spherical coordinate systems.

***Answer*:**

The conversion between the Cartesian coordinates (x, y) of a point in the plane and the polar coordinates (ρ, ϕ) of the same point is given by the relations :

$$\begin{cases} x = \rho\cos\phi \\ y = \rho\sin\phi \end{cases},$$

where $\rho \in [0, \infty)$, $\phi \in [0, 2\pi)$.

The conversion between the Cartesian coordinates (x, y, z) of a point in three-dimensional space and the cylindrical coordinates (ρ, ϕ, z) of the same point is given by the relations :

$$\begin{cases} x = \rho\cos\phi \\ y = \rho\sin\phi \\ z = z \end{cases},$$

where $\rho \in [0, \infty)$, $\phi \in [0, 2\pi)$, $\mathrm{z} \in \mathbf{R}$.

The conversion between the Cartesian coordinates (*x, y, z*) of a point in three-dimensional space and the spherical coordinates (ρ, ϕ, θ) of the same point is given by the relations :

$$\begin{cases} x = \rho\cos\phi\sin\theta \\ y = \rho\sin\phi\sin\theta \\ z = \rho\cos\theta \end{cases},$$

where $\rho \in [0, \infty)$, $\phi \in [0, 2\pi)$, $\theta \in [0, \pi]$.

10. Physical and geometrical magnitudes calculated by integrals. Formula for the flux of a vector field.

***Answer*:**

Area of a plane domain, volume of a body, mass, centre of gravity, moments of inertia, the work of a field of force.

Let S be a smooth surface and let $\vec{v} = P\vec{i} + Q\vec{j} + R\vec{k}$ be a continuous vector field on S. The flux of the vector field $\vec{v}$ across the surface S oriented by the normal vector $\vec{n} = (\cos\alpha)\vec{i} + (\cos\beta)\vec{j} + (\cos\gamma)\vec{k}$ is:

$$\iint_S (\vec{v}\vec{n})dS = \iint_S (P\cos\alpha + Q\cos\beta + R\cos\gamma)dS.$$

11. Derivative with respect to a versor of a real function. Gradient, divergence and curl.

***Answer*:**

Let $f : D \subset \mathbf{R} \to \mathbf{R}$ be a scalar field, let $\vec{s} \in \mathbf{R}^3$, ||s||=1, be a versor and $\vec{a} \in D$. The derivative of *f* in the direction of $\vec{s}$ at the point $\vec{a}$ is the limit (provided that it exists)

$$\lim_{t\to 0} \frac{1}{t}[f(\vec{a} + t\vec{s}) - f(\vec{a})] := \frac{\partial f}{\partial \vec{s}}(\vec{a})$$

The derivative $\frac{\partial f}{\partial \vec{s}}(\vec{a})$ characterizes the velocity variation of f with respect to $\vec{s}$ at the point $\vec{a}$. The gradient of f at $\vec{a}$ is defined by

$$grad f(\vec{a}) = \nabla f(\vec{a}) = \frac{\partial f}{\partial x}(\vec{a})\vec{i} + \frac{\partial f}{\partial y}(\vec{a})\vec{j} + \frac{\partial f}{\partial z}(\vec{a})\vec{k}$$

where Nabla is the operator of Hamilton: $\nabla = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$.

It can be proved that $\frac{\partial f}{\partial \vec{s}}(\vec{a}) = \vec{s} \cdot \nabla f(\vec{a})$, that is the directional derivative of f at $\vec{a}$ in the direction $\vec{s}$ is equal to the dot product between the gradient of f and $\vec{s}$.

From here it follows that the gradient direction of a scalar field is the direction of maximum value of that field, that is the field has the fastest variation.

Let $\vec{v} : U \to \mathbf{R}$ be a vector field defined on an open set $U \subset \mathbf{R}^3$, $\vec{v} = (P, Q, R)$. The divergence of the field $\vec{v}$ at a current point is the scalar (number)

$$div\vec{v} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}.$$

The curl of the field $\vec{v}$ at a current point is the vector

$$curl\vec{v} = \nabla f(\vec{a}) = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\vec{i} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\vec{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\vec{k}.$$

12. Write the Fourier series and the Fourier coefficients for a continuous periodic signal.

***Answer*:** Let $f : \mathbf{R} \to \mathbf{R}$ be an integrable and periodic function having the period T and $\omega = \frac{2\pi}{T}$. The Fourier coefficients are:

$$a_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega t)dt, \quad n = 0,1,...$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega t)dt, \quad n = 1,2,...$$

The Fourier series associated to f is:

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega x) + b_n \sin(n\omega x).$$

13. Define the Fourier transform. The Fourier inverting formula.

***Answer*:**

The Fourier transform of an absolutely integrable function $f:\mathbf{R}\to\mathbf{C}$ is:

$$\hat{f}(\omega) = \int_R f(t)e^{-it\omega}dt.$$

The Fourier inverting formula is

$$f(t) = \frac{1}{2\pi}\int_R \hat{f}(\omega)e^{it\omega}d\omega.$$

14. Write the filtering formula and the Fourier transform for the unit impulse.

***Answer*:**

The filtering formula is: $\delta(x - x_0) = \delta_{x_0}$, where δ is the Dirac's distribution.

The Fourier transform is $\hat{\delta} = 1$.

15. Solve the Cauchy-Problem

$$\begin{cases} x'(t) = a(t)x(t) \\ x(t_0) = x_0 \end{cases}$$

where a is a continuous function.

Answer:

The given equation can be rewritten as

$$\frac{x'(s)}{x(s)} = a(s).$$

Integrating between t_0 and t, we obtain

$$\ln x(t) - \ln x(t_0) = \int_{t_0}^{t} a(s)ds \Leftrightarrow \ln\frac{x(t)}{x(t_0)} = \int_{t_0}^{t} a(s)ds.$$

Thus, the sought-for solution is

$$x(t) = x_o e^{\int_{t_0}^{t} a(s)ds} \quad .$$

Physics

1. *Definition of mechanical energy*

Answer: The general form of the definition for mechanical energy is: $E_{mech} = K + PE$
Where:
PE refers to the total potential energy of the system, including all types of potential energy; [J]
K refers to the sum of the kinetic energies of all particles in the system, [J]
E_{mech} is the total mechanic energy ; [J]

2. *Definition of the kinetic energy*

Answer: The kinetic energy K of an object of mass m moving with a speed v is defined as $K = ½ (mv^2)$
K is the kinetic energy of the moving object [J]
m is the mass of the moving object [kg]
v is the speed of the object [m/s]

3. *Definition of work*

Answer: The work W done on a system by an external agent exerting a constant force on the system is the product of the magnitude F of the force, the magnitude Δr of the displacement of the point of application of the force, and cos θ,
where θ is the angle between the force and displacement vectors. The work is a scalar quantity.
W is the work [J]
F is the constant external force acting on the system [N]
Δr is the magnitude of the displacement, [m]
The work done by a variable net force is

$$\sum W = W_{\text{net}} = \int \left(\sum \vec{\mathbf{F}} \right) \cdot d\vec{\mathbf{r}}$$

where the integral is calculated over the path that the particle takes through space.

4. *Definition of potential energy*

Answer: The expression of potential energy, in linear systems, is a function of position (relative position). The corresponding force is also a function of position.
The gravitational potential energy PE is the energy that an object of mass m has by virtue of its position relative to the surface of the earth. That position is measured by the height h of the object relative to an arbitrary zero level: PE = mgh
PE is the gravitational potential energy [J]
m is the mass [kg]
g is the gravitational acceleration [m/s^2]
h is the height [m]

5. *Definition of (mechanical) power*

Answer: Power is the rate at which energy is expended or converted to another form. Mechanically, it is the rate at which work is done. Power is work done per unit time.
Average power: P = W/t = work[J]/time[s] .
SI Unit for power is the watt: 1W=1J/1s

6. Definition of heat

Answer: Heat is energy that flows from a higher-temperature object to a lower-temperature object because of the difference in temperatures. The substance has internal energy, not heat. The word "heat" is used only when referring to the energy actually in transit from hot to cold.
SI Unit of Heat: joule (J)

7. Conservation of mechanical energy

Answer: For an isolated system the energy in the system is conserved and the sum of the kinetic and potential energies remains constant. KE + PE = constant
The total mechanical energy, E_{mech} = KE + PE of an object remains constant as the object moves, provided that the net work done by external nonconservative forces is zero, $W_{nc}=0$.

8. Conservation of linear momentum (impulse) for an isolated system

Answer: The linear momentum of a particle or an object that can be modeled as a particle of mass *m* moving with a velocity $\vec{v}$ is defined to be the product of the mass and velocity: $\vec{p} = m\vec{v}$.
The total linear momentum of an isolated (net external force equal to zero) system remains constant.

$$\sum \vec{F} = \frac{d\overline{p}}{dt} = 0; \qquad \vec{p}_{tot} = \text{constant}$$

9. The conservation of the angular momentum

Answer: The instantaneous angular momentum of the particle relative to the origin *O* is defined by the vector $\overline{L}$
product of its instantaneous position vector $\overline{r}$ and the instantaneous linear momentum $\overline{p}$.

$$\overline{L} = \overline{r} \times \overline{p}$$

The total angular momentum of a system is conserved if the net external torque acting on the system is zero.

$$\overline{M}_{ext} = \overline{r} \times \overline{F_{ext}} \qquad \overline{M} = \frac{d\overline{L}}{dt} \qquad \text{External } \overline{M} = 0, \rightarrow \frac{d\overline{L}}{dt} = 0, \rightarrow \overline{L} = \text{constant}$$

10. The Hooke's law

Answer: The force of an elastic system (spring), inside the limits of linearity (elasticity) is given by

$$\vec{F} = -k\vec{x}$$

Where x is the displacement of the spring's end from its equilibrium position (a distance, in SI units: m);

F is the restoring force exerted by the material (in SI units: N or $kgms^{-2}$); and
k is a constant called the *rate* or *spring constant* (in SI units: $N \cdot m^{-1}$ or kgs^{-2}).

For an elastic bar: $$\Delta l = \frac{F \cdot l_0}{S \cdot E}$$

Where F is the force [N] , l_0 is the initial length of the bar [m], S is the cross section of the bar [m^2] and E is the Young' module (elasticity) of the material of the bar [N/m^2].

11. Archimedes's law

Answer: The apparent loss in weight of a body immersed in a fluid is equal to the weight of the displaced fluid
Or: a body immersed in a fluid is pushed up, in the vertical direction, with a force equal to the weight of the volume of the displaced fluid.

12. The law of absorption of waves

A: In a homogenous dissipative media the intensity of plane waves reduces exponentially with the distance

$$I = I_0 e^{-kx}$$

where I_0 is the intensity of the penetrating wave, I is the intensity of the wave at distance x, and k is the absorption coefficient.
The absorption coefficient is a characteristic of the medium, depending also on the wave length of the incident wave
The intensity „I" of the wave is numerically equal to the energy carried by the wave in a second, trough the surface normal (orthogonal on the wave direction of propagation.

13. The reflection laws

Answer: The incident ray, the reflected ray, and the normal to the surface all lie in the same plane, and
the angle of reflection equals the angle of incidence .

14. The refraction laws

Answer: When light travels from a material with refractive index n_1 into a material with refractive index n_2, the refracted ray, the incident ray, and the normal to the interface between the materials all lie in the same plane. The angle of refraction is related to the angle of incidence by $n_1 \cdot \sin\theta_1 = n_2 \cdot \sin\theta_2$.

The index of refraction n of a material is the ratio of the speed c of light in a vacuum to the speed v of light in the material.

15. Coulomb's law

Answer: The magnitude F of the electrostatic force exerted by one point charge q_1 on another point charge q_2 is directly proportional to the magnitudes $|q_1|$ and $|q_2|$ of the charges and inversely proportional to the square of the distance r between them.

$$F = \frac{q_1 \cdot q_2}{4\pi\varepsilon} \cdot \frac{1}{r^2}$$

The electrostatic force is directed along the line joining the charges, and it is attractive if the charges have unlike signs and repulsive if the charges have like signs.

Measuring Units

of the International System of Units

1. ***Specify the SI unit and its symbol for mass. Specify the multiplier and its symbol for micro (example: atto = 10^{-18}, a).***

   The SI unit for mass is the kilogram. Its symbol is kg. The multiplier for micro is 10^{-6}. Its symbol is μ.

2. ***Specify the SI unit and its symbol for length. Specify the multiplier and its symbol for milli (example: atto = 10^{-18}, a).***

   The SI unit for length is the metre. Its symbol is m. The multiplier for milli is 10^{-3}. Its symbol is m.

3. ***Specify the SI unit and its symbol for time. Specify the multiplier and its symbol for micro (example: atto = 10^{-18}, a).***

   The SI unit for time is the second. Its symbol is s. The multiplier for micro is 10^{-6}. Its symbol is μ.

4. ***Specify the SI unit and its symbol for electrical current. Specify the multiplier and its symbol for milli (example: atto = 10^{-18}, a).***

   The SI unit for electrical current is the ampere. Its symbol is A. The multiplier for milli is 10^{-3}. Its symbol is m.

5. ***Specify the SI unit and its symbol for angular velocity. Specify the multiplier and its symbol for kilo (example: atto = 10^{-18}, a).***

   The SI unit for angular velocity is the radian per second. Its symbol is rad/s. The multiplier for kilo is 10^{3}. Its symbol is k.

6. ***Specify the SI unit and its symbol for frequency. Specify the multiplier and its symbol for tera (example: atto = 10^{-18}, a).***

   The SI unit for frequency is the hertz. Its symbol is Hz. The multiplier for tera is 10^{12}. Its symbol is T.

7. ***Specify the SI unit and its symbol for energy, work and heat. Specify the multiplier and its symbol for mega (example: atto = 10^{-18}, a).***

   The SI unit for energy, work and heat is the joule. Its symbol is J. The multiplier for mega is 10^{6}. Its symbol is M.

8. ***Specify the SI unit and its symbol for power and radiant flux. Specify the multiplier and its symbol for giga (example: atto = 10^{-18}, a).***

The SI unit for power and radiant flux is the watt. Its symbol is W. The multiplier for giga is 10^9. Its symbol is G.

9. Specify the SI unit and its symbol for electrical charge and quantity of electricity. Specify the multiplier and its symbol for femto (example: atto = 10^{-18}, a).

The SI unit for electrical charge and quantity of electricity is the coulomb. Its symbol is C. The multiplier for femto is 10^{-15}. Its symbol is f.

10. Specify the SI unit and its symbol for voltage, electrical potential difference and electromotive force. Specify the multiplier and its symbol for nano (example: atto = 10^{-18}, a).

The SI unit for voltage, electrical potential difference and electromotive force is the volt. Its symbol is V. The multiplier for nano is 10^{-9}. Its symbol is n.

11. Specify the SI unit and its symbol for electrical field strength. Specify the multiplier and its symbol for mega (example: atto = 10^{-18}, a).

The SI unit for electrical field strength is the volt per metre. Its symbol is V/m. The multiplier for mega is 10^6. Its symbol is M.

12. Specify the SI unit and its symbol for electric resistance, impedance and reactance. Specify the multiplier and its symbol for kilo (example: atto = 10^{-18}, a).

The SI unit for electric resistance, impedance and reactance is the ohm. Its symbol is Ω. The multiplier for kilo is 10^3. Its symbol is k.

13. Specify the SI unit and its symbol for electrical conductance. Specify the multiplier and its symbol for kilo (example: atto = 10^{-18}, a).

The SI unit for electrical conductance is the siemens. Its symbol is S. The multiplier for kilo is 10^3. Its symbol is k.

14. Specify the SI unit and its symbol for electric capacitance. Specify the multiplier and its symbol for pico (example: atto = 10^{-18}, a).

The SI unit for electric capacitance is the farad. Its symbol is F. The multiplier for pico is 10^{-12}. Its symbol is p.

15. Specify the SI unit and its symbol for inductance. Specify the multiplier and its symbol for milli (example: atto = 10^{-18}, a).

The SI unit for inductance is the henry. Its symbol is H. The multiplier for milli is 10^{-3}. Its symbol is m.

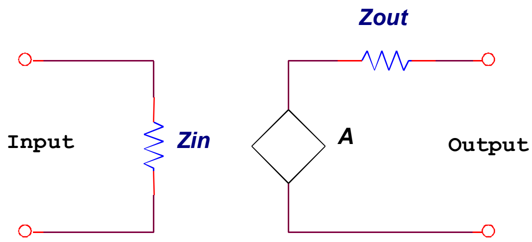
Electronic Circuits

1. *Demonstrate the optimum input and output impedance for a voltage amplifier.*

2011 EC (c 01).ppt / slides 8,9

Amplifiers fundamental properties

- *Gain*
- *Input impedance*
- *Output impedance*



General amplifier model

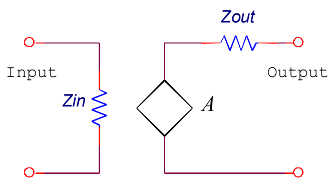
- There are three types of gain:

voltage gain (A_V), *current gain* (A_I), *power gain* (A_P).

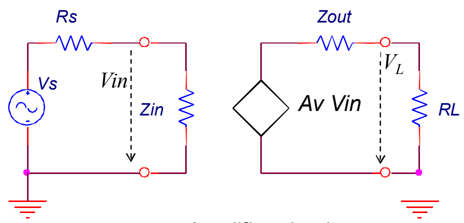
$$A_V = \frac{V_{OUT}}{V_{IN}} \qquad A_I = \frac{I_{OUT}}{I_{IN}} \qquad A_P = \frac{P_{OUT}}{P_{IN}}$$

- The gain of a circuit is determined by its component values !!!
- When the **gain** of a circuit has been calculated, it can be used to determine the **output** of the circuit for **a specified input**

Ex: Voltage amplifier circuit



Amplifier model



Amplifier circuit

- At the circuit input and output there are 2 voltage dividers:

$$v_{IN} = v_S \frac{Z_{IN}}{R_S + Z_{IN}} \qquad v_L = v_{OUT} \frac{R_L}{Z_{OUT} + R_L} \qquad \text{where} \quad v_{OUT} = A_V v_{IN}$$

- Since $v_{IN} < v_S$ and $v_L < v_{OUT}$

=> The *effective voltage gain* of a circuit is lower than the calculated voltage gain of the amplifier itself.

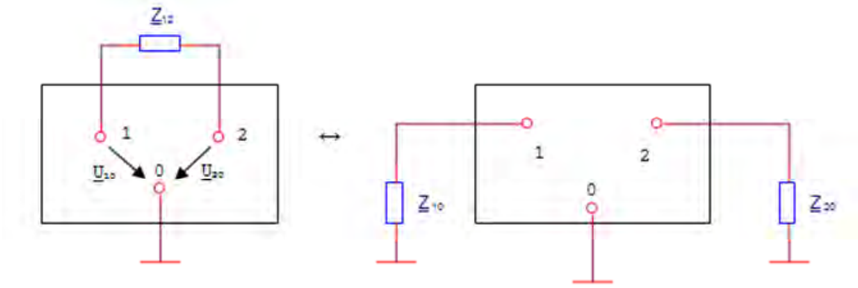
To neglect the input and output voltage drop we must have :

[Infinite gain], Infinite input impedance, Zero output impedance !!!

2. ***Explain Miller effect and theorem and its utility for high frequency analysis.***
[2011 EC (c 03+04).ppt] /slides 37-38, [seminar nr.2.doc]

Miller Effect

An impedance Z_{12} connected from the input of an amplifier to the output can be replaced by an impedance across the input terminals (Z_{10}) and impedance across the output terminals (Z_{20}).



Miller Effect

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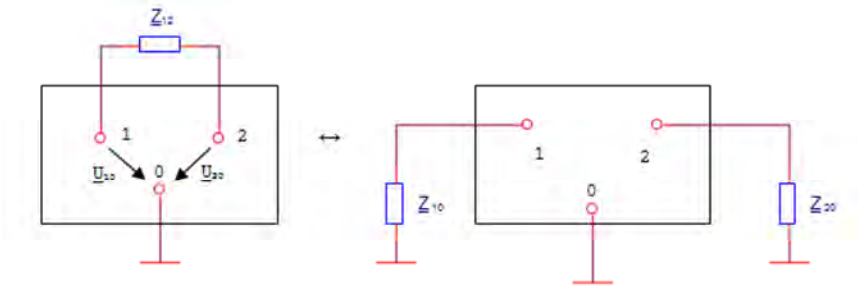
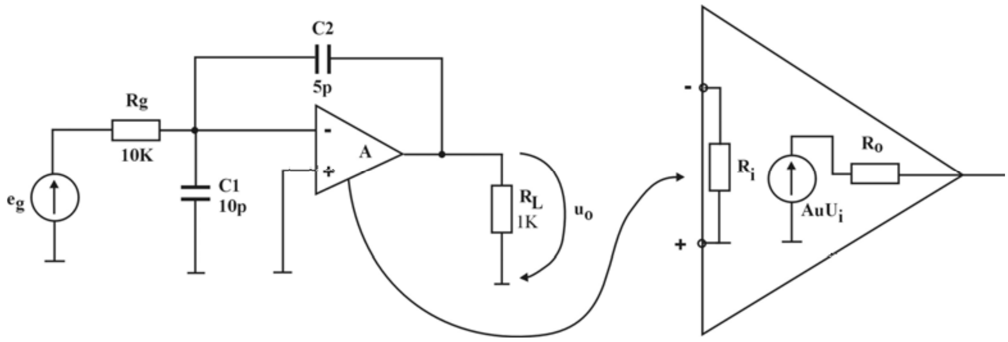


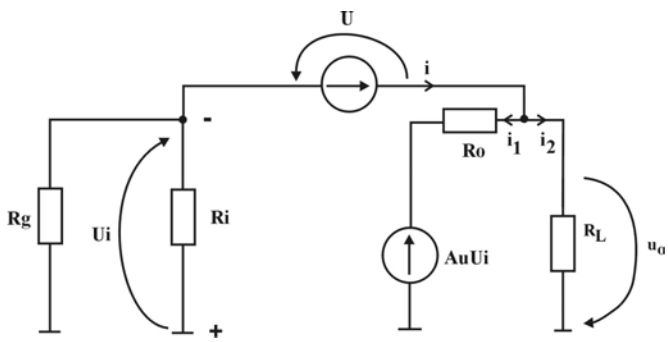
Example: For the following circuit, we consider the amplifier's parameters:

$A_u = 10^2$, $R_i = 1M\Omega$, $R_0 = 1K$.



Compute high cut of limit frequency of the circuit, using Miller's theorem

Answer:



$$f_{p_1} = \frac{1}{2\pi(C_{10} + C_1)R_g \| R_i} = 60KHz \quad , \quad f_{p_2} = \frac{1}{2\pi(C_{20})R_g \| R_i} = 64KHz$$

$$C_{10} = C_2(1 - K),\ C_{20} = C_2(1 - \frac{1}{K}),\ \ K = \frac{U_o}{U_i} = \frac{A_u U_i \dfrac{R_L}{R_o + R_L}}{U_i} = A_u \frac{R_L}{R_o + R_L}$$

$f_p2 >> f_{p1} \rightarrow$ $\quad f_i \cong f_{p1} = 60\text{KHz}$

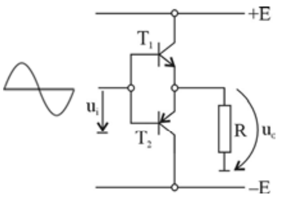
3. ***Which amplifier class is known for its crossover distortions? Explain the root cause and ways of improvement based on a simplified schematic and is transfer characteristic.***

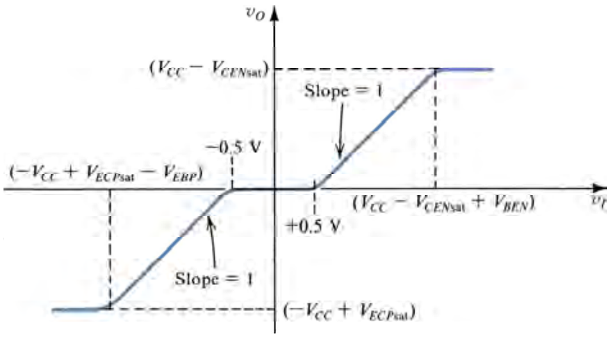
2011 EC (c 05).ppt / slides 22-24, 37

Class B - output stage circuit

Named also *Complementary-symmetry amplifier*

= 2 complementary BJT's used as emitter followers, working in "push pull" mode.





Advantage: good efficiency – up to 78.5% (in DC mode – no current sink from supply)

Drawback: Crossover distortion

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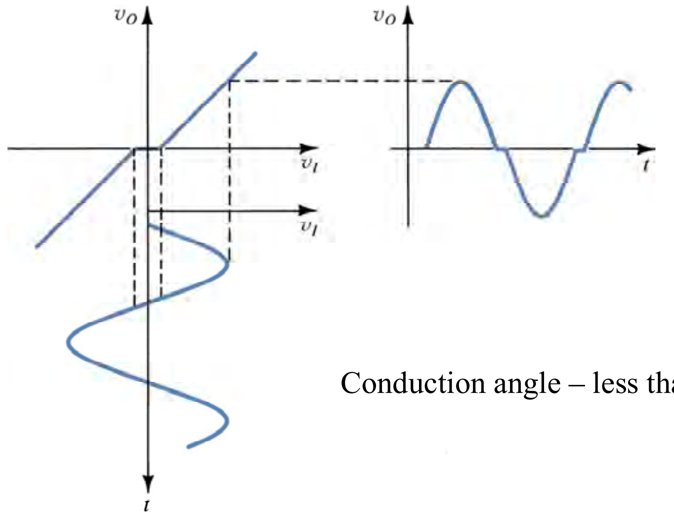
Class B – crossover distortion

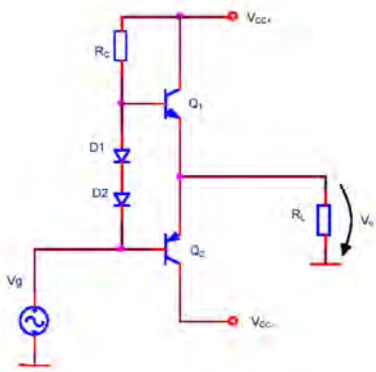


Conduction angle – less than ½ T

=> crossover distortion between the “halves” of the signal

Class B biasing

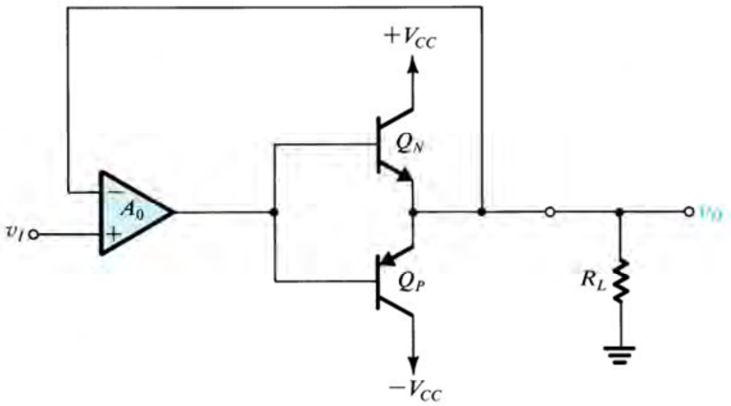
Biasing must provide V(B1)- V(B2) to keep Q1 and Q2 off, but close to conduction => lower crossover distortion



BJT ‘s can be biased using 2 diodes or a “V_{BE} multiplier” circuit => constant voltage drop between Q1 and Q2 bases

Opamp Implementation

Op amp connected in a negative-feedback loop to reduce crossover distortion



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4. *Describe half bridge class D amplifiers topology, block schematic and principle of operation.*

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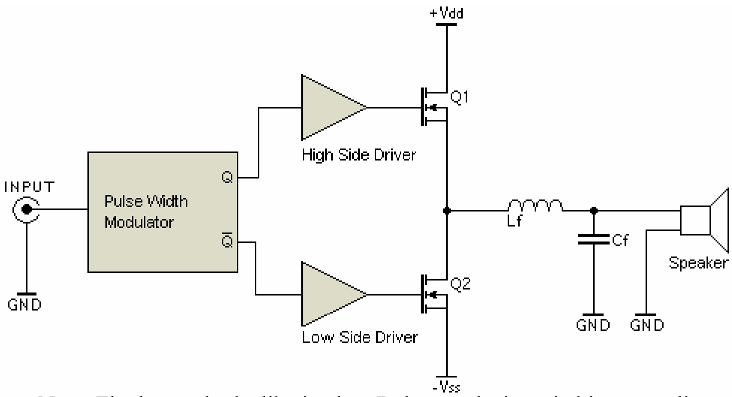
Class D – (half bridge) simplified circuit

– operation is switching, hence the term *switching power amplifier*
– output devices are rapidly switched on and off at least twice for each cycle
– the output devices are either completely on or completely off so theoretically they do not dissipate any power



Note: Final stage looks like in class B, but works in switching – not linear mode !!!

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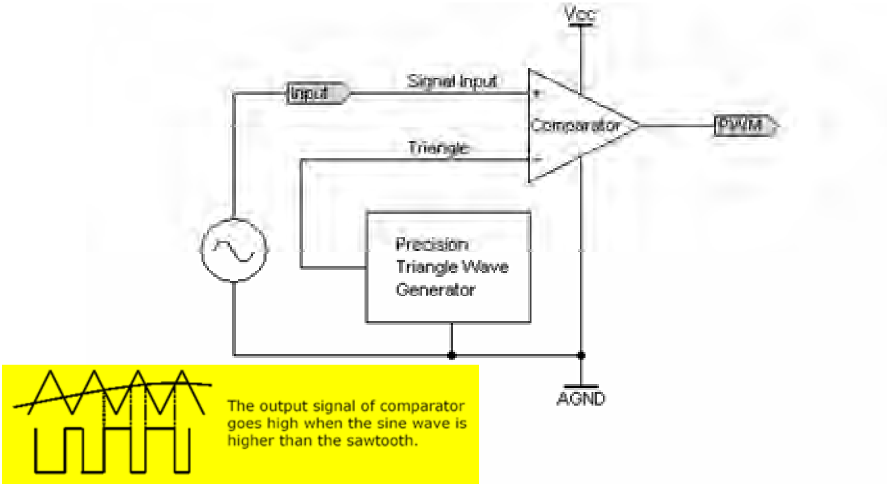
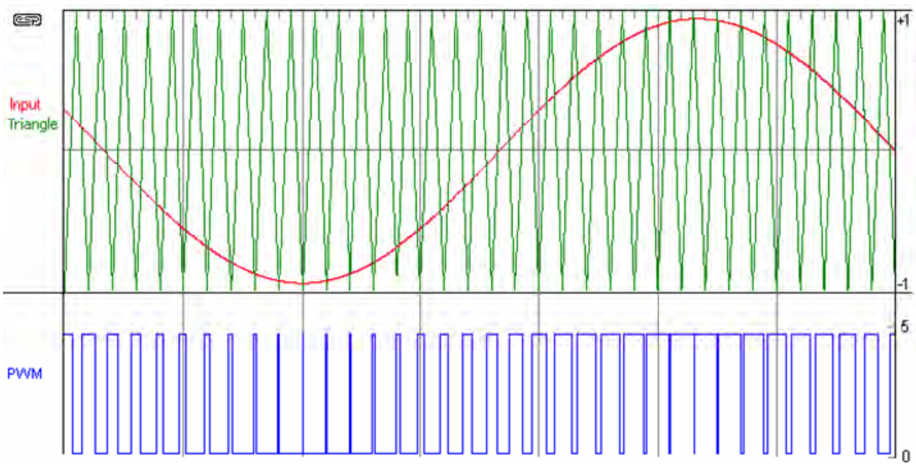
Class D - PWM signal generation

▪The input signal is compared with a triangle signal resulting in a PWM (Pulse width modulation) signal



Class D – PWM waveforms



▪Usually 150KHz to 250Khz switching freq. is used

▪The LC LPF provide at the output the mean value of the PWM signal - same shape as the Input signal

5. Using formula show the most unwanted occurrence place for an external perturbation in a multistage amplifier sing a gobal negative feedback loop

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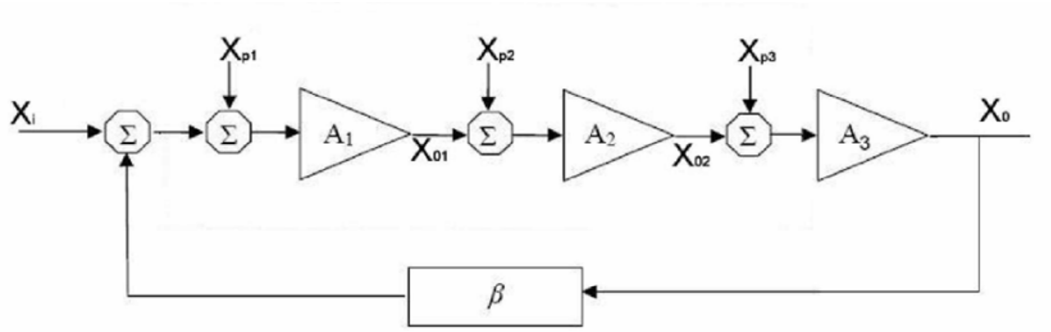
Perturbation influence in Feedback Amps



$$\begin{cases} x_0 = (x_{02} + x_{p3})A_3 \\ x_{02} = (x_{01} + x_{p2})A_2 \\ x_0 = (x_\Sigma + x_{p1})A_1 \\ x_\Sigma = x_i - x_r = x_i - \beta x_0 \end{cases}$$

$$x_0 = (((x_i - \beta x_0 + x_{p1})A_1 + x_{p2})A_2 + x_{p3})A_3$$

Perturbations are reduced as they are closer to output

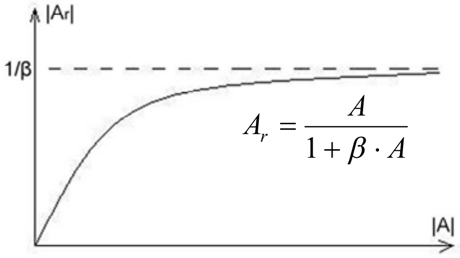
$$\Rightarrow x_0 = x_i \frac{A_1 A_2 A_3}{1 + \beta A_1 A_2 A_3} + x_{p1} \frac{A_1 A_2 A_3}{1 + \beta A_1 A_2 A_3} + x_{p2} \frac{A_2 A_3}{1 + \beta A_1 A_2 A_3} + x_{p3} \frac{A_3}{1 + \beta A_1 A_2 A_3}$$

6. *Demonstrate bandwidth extension for an amplifier when a negative feedback is applied.*

2011 EC (c 07).ppt /slides 5-7

Feedback effect over gain

- For amplifiers with feedback we can assume that the



If nonfeedback amplifier has : $\Delta A / A$

???

$$\Delta A_r / A_r \longleftrightarrow \Delta A / A$$

At small variations :

$$\Delta A_r \cong \frac{1+\beta A - A\beta}{(1+\beta A)^2} \cdot \Delta A = \frac{1}{(1+\beta A)^2} \cdot \Delta A = \frac{A}{1+\beta A} \cdot \frac{1}{1+\beta A} \cdot \frac{\Delta A}{A} \Rightarrow$$

$$\frac{\Delta A_r}{A_r} \cong \frac{\Delta A}{A} \cdot \frac{1}{1+\beta A} = \frac{\Delta A}{A} \cdot \frac{1}{F}$$

F times improvement !!!

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Influence of the feedback on freq. response

If $A(j\omega) = \frac{P(j\omega)}{Q(j\omega)} \cdot A_0$ and $\beta = \beta_0$ a real number,

Then: $$A_r(j\omega) = \frac{\frac{P}{Q} \cdot A_0}{1+\beta_0 \frac{P}{Q} \cdot A_0} = \frac{PA_0}{Q+\beta_0 PA_0}$$

only the poles are shifted

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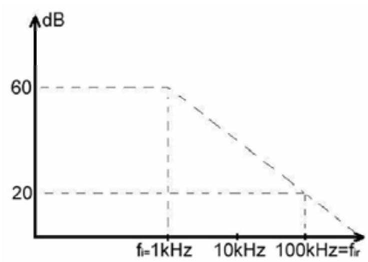
Influence of the feedback on freq. response(@high freq.)

$$\text{EX 1}: \; A(j\omega) = \frac{A_0}{1 + j\frac{f}{f_i}} \qquad \beta = \beta_0$$

$$A_r(j\omega) = \frac{\frac{A_0}{1+j\frac{f}{f_i}}}{1 + \frac{\beta_0 A_0}{1+j\frac{f}{f_i}}} = \frac{A_0}{1+\beta_0 A_0 + j\frac{f}{f_i}} = \frac{A_0}{1+\beta_0 A_0}\cdot\frac{1}{1+j\frac{f}{f_i(1+\beta_0 A_0)}} = A_{ro}\cdot\frac{1}{1+j\frac{f}{f_{ir}}}$$

where $A_{r0} = \frac{A_0}{1+\beta_0 A_0}$ & $f_{ir} = f_i(1+\beta_0 A_0)$

Then: $A_r = \frac{A}{F}$; $f_{ir} = f_i F$; $A_r \cdot f_{ir} = A \cdot f_i$



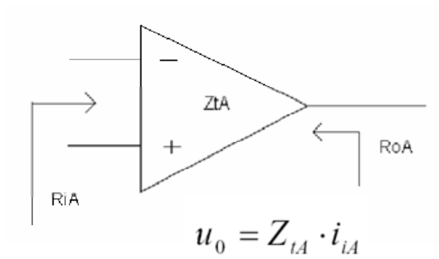
Note: only if circuit still behave linear !!!

7. Show input and output resistance change for an amplifier when a shunt-shunt feedback is applied. Justify with formulas.

2011 EC (c 08).ppt / slides 7,8,10

Shunt Shunt Negative Feedback

$$\begin{cases} u_0 = Z_t \cdot \frac{R_L \| R_{if}}{R_L \| R_{if} + R_0} \cdot i_i \\ i_i = i_{iA} \cdot \frac{R_g \| R_{of}}{R_g \| R_{of} + R_i} \\ i_{iA} = i_g - i_r = i_g - \beta u_0 \end{cases}$$



$$u_0 = Z_{tA} \cdot i_{iA}$$

$$Z_{tA} = \frac{u_0}{i_g}\Big|_{\beta=0}$$

$$\beta = \frac{i_r}{u_0}\Big|_{u_i=0}$$

-Obtained by separating feedback network

$$u_0 = \underbrace{Z_t \cdot \frac{R_L \| R_{if}}{R_L \| R_{if} + R_0} \cdot \frac{R_g \| R_{of}}{R_g \| R_{of} + R_i}}_{Z_{tA}} \cdot i_{iA}$$

Transfer impedance of the amp. with influences included and no feedback

Shunt Shunt Negative Feedback

$$Z_{tA} = Z_{tr}|_{\ =0}$$

$$u_0 = Z_{tA} \cdot i_{iA} = Z_{tA}(i_g - i_r) = Z_{tA}(i_g - \beta u_0)$$

$$=> \quad \boxed{Z_{tr} = \frac{u_0}{i_g} = \frac{Z_{tA}}{1 + \beta \cdot Z_{tA}}}$$

$$R_{ir} = \frac{u_i}{i_g}$$

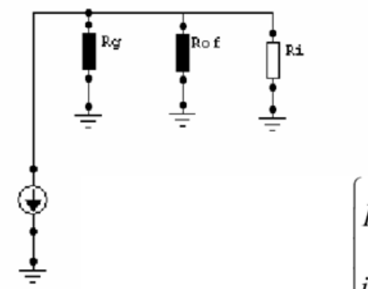
$$i_g = i_r + i_{iA} = \beta u_0 + \frac{u_i}{\underbrace{R_g \| R_{of} \| R_i}_{R_{iA}}} = \beta u_0 + \frac{u_i}{R_{iA}} = \beta \cdot Z_{tA} \cdot i_{iA} + \frac{u_i}{R_{iA}}$$

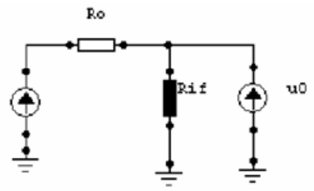
$$i_g = \beta \cdot Z_{tA} \cdot \frac{u_i}{R_{iA}} + \frac{u_i}{R_{iA}} \quad => \quad \boxed{R_{ir} = \frac{u_i}{i_g} = \frac{R_{iA}}{1 + \beta \cdot Z_{tA}}}$$

Small value, because ig split also to the feedback network.

Shunt Shunt Negative Feedback

The output resistance determination





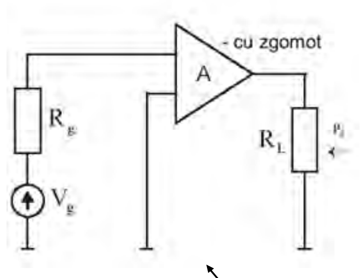
$$R_{or} = \frac{u_0}{i_0} | \, i_g{=}0$$

$$\begin{cases} R_{or} = \dfrac{u_0}{i_0}\Big|_{i_g} = 0 \\ i_{iA} = -\beta u_0 \\ i_0 = \dfrac{u_0}{R_{if}} + \dfrac{u_0 - Z_t \cdot i_i}{R_0} \Rightarrow i_0 = \dfrac{u_0}{R_{if}} + \dfrac{u_0(1+\beta Z_t)}{R_0} \\ i_i = i_{iA} \cdot \dfrac{R_g \| R_{of}}{R_g \| R_{of} + R_i} \end{cases}$$

8. Draw noise equivalent schematic of an amplifier and define noise factor F.

2011 EC (c 11).ppt / slides 17, 18, 19

Noise model for an Amplifier

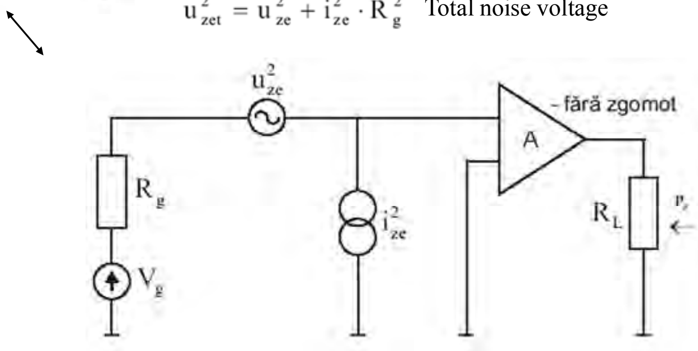


If the noise sources are uncorrelated, noise at output:

$$P_z = \sum_{k=1}^{n} P_{zk}$$

$$P_{zk} \rightarrow i_{zk}^2, u_{zk}^2 \rightarrow R_L$$

$$u_{zet}^2 = u_{ze}^2 + i_{ze}^2 \cdot R_g^2$$ Total noise voltage



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Noise figure (factor)

- Compare noise produced by the amp. with the noise produced by the generator Rg

$$F = \frac{SNR_{in}}{SNR_{out}}$$ SNR – signal to noise ratio

$$F \stackrel{def}{=} \frac{\text{Puterea totală de zgomot de la iesire}}{\text{Puterea de zgomot datorată generatorului}}$$

$$F_{dB} = 10 \lg F$$

$$F_{dB} = 10 \lg \frac{P_{zg} + P_{zA}}{P_{zg}} \text{ unde } P_{zg} = \frac{u_{zg}^2}{(R_g + R_i)^2} \cdot R_i^2$$

$$P_{zA} = \frac{u_{ze}^2}{(R_g + R_i)^2} \cdot R_i^2 + \frac{i_{ze}^2 \cdot R_g^2}{(R_g + R_i)^2} \cdot R_i^2$$

$$F_{dB} = 10 \lg \left(1 + \frac{u_{ze}^2 + i_{ze}^2 \cdot R_g^2}{u_{zR_g}^2} \right) \qquad u_{zR_g}^2 = 4kTR_g \cdot \Delta f$$

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9. Explain dominant pole (lag) compensation method. How is related dominant pole frequency to gain unity frequency f_{0dB}. Show practical implementation.

2011 EC (c 10).ppt / slides 19-21

4.1 Dominant-pole compensation

It represents a very popular method, also called **lag compensation**. It consists in adding another pole in the open-loop transfer function - A(jω) - at a very low frequency, such that the loop-gain drops to unity by the time the phase reaches -180°:

$$A_C(j\omega) = A(j\omega)\frac{1}{1+j\dfrac{f}{f_d}}$$

$$f_d << \min(f_{pk})$$

where f_{pk} are the pole frequencies for A(jω).

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A serious disadvantage of this compensation method is the resulting close-loop amplifier bandwidth, drastically reduced (fig. 4).

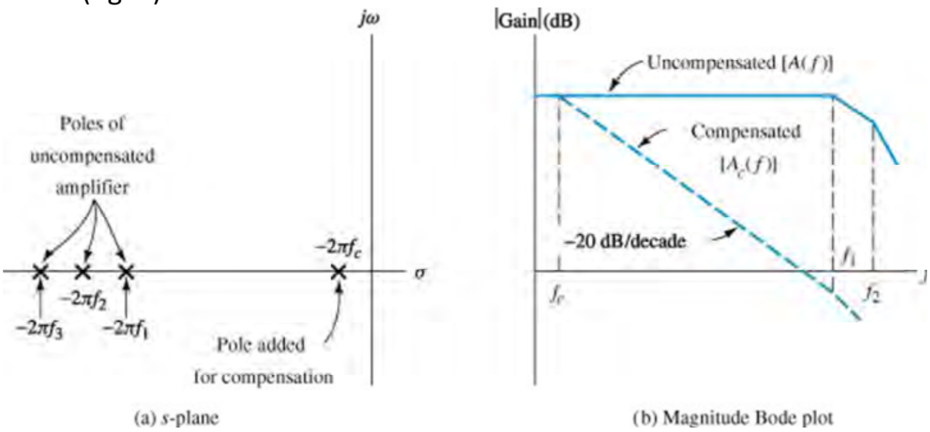

Fig. 4. Dominant-pole compensation.

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